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ELASTOPLASTIC FILTRATION OF LIQUID IN UNSTABLE SEAMS

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Equations of elastoplastic filtration of liquid in seams, whose skeleton can fail under large effective stresses, have been derived. On the basis of their numerical solution the influence of the seam instability factor on the filtration characteristics has been established.

Although the theory of elastoplastic filtration of liquids was developed relatively long ago, today it is gaining in importance in connection with the exploration and development of deep oil and gas fields in the majority of oil- and gas-extracting regions of the world. In so doing, there occur sharp irreversible decreases in the productivity of operating holes with decreasing hole bottom and seam pressure on the whole [1, 2], which leads to a strengthening of the stressed-strained state of the seam skeleton, skeleton particle motion, and a change in the structure of the cementing material [1].

Because of the nonuniformity of their mechanical properties, in the rocks of oil-containing seams-traps some of the particles are in the state of elastic strain and others, under the same conditions, are in the state of plastic strain. Upon recovery of the seam pressure, some particles, being elastic, try to restore their volume and shape, whereas others retain the acquired strain partially or completely. Many researchers [1] point to the fact that the degree of residual strain depends on the composition of the liquid saturating the rock. Hence it is apparent that liquid filtration in the rocks of the seam can occur under both its elastic and plastoelastic and plastic strains.

The first approach to the construction of mathematical models of plastoelastic filtration in porous media was used in [3, 4]. This theory was further developed in [1, 5, 6].

In deep-lying oil pools, due to the growth of effective stresses the plastoelastic regime of their strain takes place, and under large effective stresses the pool skeleton may become destroyed. In other words, the pool loses its stability. High rock pressures lead to the fact that the seam roof is subjected to a considerable deformation and its bending around the hole is observed. Due to the destruction of the seam skeleton a zone is formed around the hole where the seam-skeleton particles are brought by a fluid flow to the holes and get onto the surface. Therefore, in seams where the rocks are formed from weakly cemented particles, under large depressions on the seam, plastoelastic filtration with a simultaneous breaking of its integrity is observed. No rigorous theory of the filtration process in the plastoelastic regime that takes into account the instability of the seam skeleton, i.e., its destruction, has been advanced up to now. Some phenomenological approaches to the modeling of this phenomenon were used in [7, 8] and individual standard problems were considered on their basis [9, 10]. Note that in [7, 8] some phenomenological parameters characterizing the intensity of the change in the seam permeability and porosity due to the skeleton destruction and particles being carried out of the skeleton were introduced. Obviously, a more rigorous approach should be based on the analysis of the stressed-strained state of the seam around the hole in the process of the pressure decrease and its subsequent recovery. In so doing, the seam destruction should be determined on the basis of the fundamental relations of mechanics of the solid deformed body. However, as will be seen from the further expounding, the use of the phenomenological approach yields many new results that are useful for analyzing the processes proceeding under plastoelastic filtration of liquids in unstable seams.

Following [7], below we first derive the equations of plastoelastic filtration in unstable seams taking into account the carrying-out of destroyed particles of the seam skeleton. Then these equations are solved numerically, under

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the conditions of decreasing and increasing pressure, for certain operating conditions of the oil pool. Calculations of the permeability, porosity, and pressure fields for the Namangan (Uzbekistan) oil deposit have been made. Conclusions concerning the influence of the seam-skeleton destruction and the particles being carried out of the seam on the filtration characteristics under plastoelastic conditions have been drawn.

The dependences of porosity m and permeability k on the effective pressure have the form of curves convex to the axes k/k_0 and m/m_0. Theoretically, these dependences can be described by different mathematical models. In [5, 11], it has been shown that such curves are described with a high degree of accuracy in a wide pressure range by the exponential dependences

$$k = k_0 \exp(-a_{k0}(p_0 - p)) , \quad m = m_0 \exp(-\beta_{m0}(p_0 - p)) . \tag{1}$$

The change in the viscosity and density of oil and water as a function of pressure is well described by the exponential dependence

$$\mu = \mu_0 \exp(-a_\mu (p_0 - p)) , \quad \rho = \rho_0 \exp(-\beta_{\text{liq}} (p_0 - p)) . \tag{2}$$

In the regime where the seam pressure decreases (↓) from the initial pressure p_0 to the current pressure p in stable seams, the filtration equation is of the form [12, 13]

$$\downarrow \frac{\partial \varphi}{\partial t} = D^2 \Delta \varphi^\gamma , \tag{3}$$

where

$$\varphi = \exp[-\beta (p_0 - p)] ; \quad D^2 = k_0/(\mu_0 m_0 \alpha) ; \quad \beta = \beta_{\text{liq}} + \beta_{m0} ; \quad \alpha = \beta_{\text{liq}} - a_\mu + a_{k0} ; \quad \gamma = \alpha/\beta .$$

When the pressure recovers (↑), the liquid filtration equation in the plastoelastic regime with no allowance for the seam-destruction factor in the one-dimensional case is of the form [1, 6]

$$\uparrow \frac{\partial}{\partial t} \left\{ \exp[-\varphi_2(x)] \exp[-\psi_2(x)(p_0 - p)] \right\} = \frac{k_0}{m_0 \rho_0} \frac{\partial}{\partial x} \left\{ \exp[-\varphi_1(x)] \exp[-\psi_1(x)(p_0 - p)] \frac{\partial p}{\partial x} \right\} , \tag{4}$$

Here

$$\varphi_1(x) = a_{k0} \left[1 - \psi(x)^{\eta_k/\beta} \right] \left[-\frac{1}{\beta} \ln |\psi(x)| \right] ; \quad \varphi_2(x) = \beta_{m0} \left[1 - \psi(x)^{\eta_m/\beta} \right] \left[-\frac{1}{\beta} \ln |\psi(x)| \right] ;$$

$$\psi_1(x) = \beta_{\text{liq}} - a_\mu + a_{k0} \psi(x)^{\eta_k/\beta} ; \quad \psi_2(x) = \beta_{\text{liq}} + \beta_{m0} \psi(x)^{\eta_m/\beta} ,$$

and $\psi(x) = \exp[-\beta(p_0 - p_1)]$ is the solution of (3) at the last moment of the pressure-decrease phase.

Suppose that in the pressure-decrease regime $p_0 \ge p \ge p_1$ the seam is broken due to the increase in the effective pressure, which leads to the carrying-out of particles separated from the rock skeleton. Denote by p_s the pressure at which this regime begins. It is clear that $p_0 \ge p_s \ge p_1$. Then, beginning with $p \le p_s$, dependences (1) should be modified. Assume that in the process of skeleton destruction and particles being carried out of the seam k and m increase and this increase is exponential. Then the total change in k and m can be written in the form

$$\begin{aligned} \downarrow k &= k_0 \exp(-a_{k0}(p_0 - p)) + \theta(p_s - p) k_{s0} [1 - \exp(-a_{ks}(p_s - p))] , \\ \downarrow m &= m_0 \exp(-\beta_{m0}(p_0 - p)) + \theta(p_s - p) m_{s0} [1 - \exp(-\beta_{ms}(p_s - p))] . \end{aligned} \tag{5}$$

Substituting (5) and (2) into the continuity equation

$$\frac{\partial (m\rho)}{\partial t} + \text{div}(\rho \mathbf{w}) = 0 \tag{6}$$

and using the Darcy law

$$\mathbf{w} = -\frac{k}{\mu} \text{grad}\, p\,, \tag{7}$$

where **w** is the filtration speed vector, in the one-dimensional case we have

$$\downarrow \frac{\partial}{\partial t}\left[\varphi + \frac{m_{s0}\theta(\sigma(\varphi))}{m_0}(1 - \delta_1 \varphi^{\beta_{ms}/\beta})\, \varphi^{\beta_{\text{liq}}/\beta}\right] = \chi_1 \frac{\partial}{\partial x}\left\{\left[\varphi^{\gamma-1} + \frac{k_{s0}\,\theta(\sigma(\varphi))}{k_0}[1 - \delta_2 \varphi^{a_{ks}/\beta}]\varphi^{-(a_\mu + \beta_{m0})/\beta}\right]\frac{\partial \varphi}{\partial x}\right\}. \tag{8}$$

Here

$$\delta_1 = \exp[-\beta_{ms}(p_0 - p_s)]\,, \quad \delta_2 = \exp[-a_{ks}(p_0 - p_s)]\,, \quad \sigma(\varphi) = p_s - p_0 - (1/\beta)\ln\varphi\,, \quad \chi_1 = k_0/(\mu_0 m_0 \beta)\,.$$

Under the conditions

$$a_{ks}(p_s - p) << 1\,, \quad \beta_{ms}(p_s - p) << 1\,, \quad p \le p_s\,,$$

in (5), instead of the exponential dependences, in the second terms on the right-hand side we can use the linear dependences

$$\begin{aligned} &\downarrow k = k_0 \exp(-a_{k0}(p_0 - p)) + \theta(p_s - p)\, k_{s0} a_{ks}(p_s - p)\,, \\ &\downarrow m = m_0 \exp(-\beta_{m0}(p_0 - p)) + \theta(p_s - p)\, m_{s0}\beta_{ms}(p_s - p)\,. \end{aligned} \tag{9}$$

Using (9) instead of (5), we obtain the filtration equation

$$\downarrow \frac{\partial}{\partial t}\left[\varphi + \frac{m_{s0}(\theta(\sigma))}{m_0}\beta_{ms}\left(p_s - p_0 - \frac{1}{\beta}\ln\varphi\right)\varphi^{\beta_{\text{liq}}/\beta}\right] =$$

$$= \chi_1 \frac{\partial}{\partial x}\left\{\left[\varphi^{\gamma-1} + \frac{k_{s0}\theta(\sigma(\varphi))}{k_0} a_{ks}\left(p_s - p_0 - \frac{1}{\beta}\ln\varphi\right)\varphi^{-(a_\mu + \beta_{m0})/\beta}\right]\frac{\partial\varphi}{\partial x}\right\}. \tag{10}$$

As is seen from (10), the replacement of (5) by (9) practically does not simplify the filtration equation.

When the seam pressure is recovered, the terms of (5) remain unchanged with time with $p = p_1$, and the decrease in k and m due to the increase in the pressure is determined by the formula

$$\begin{aligned} &\uparrow k = k_0 \exp(-(a_{k0} - a_{k1})(p_0 - p_1)) \exp(-a_{k1}(p_0 - p)) + \theta(p_s - p_1)\, k_{s0}[1 - \exp(-a_{ks}(p_s - p_1))]\,, \\ &\uparrow m = m_0 \exp(-(\beta_{m0} - \beta_{m1})(p_0 - p_1)) \exp(-\beta_{m1}(p_0 - p)) + \theta(p_s - p_1)\, m_{s0}[1 - \exp(-\beta_{ms}(p_s - p_1))]\,. \end{aligned} \tag{11}$$

Let us assume that in the regime of pressure decrease the seam operated till $t = T$ and the pressure distribution $p(T, x) = p_1(x)$ has become steady. Then $\varphi(t, x)$ we have

$$\varphi(T, x) = \exp[-\beta(p_0 - p_1)] = \psi(x)\,,$$

hence

$$p_0 - p_1 = -(1/\beta)\ln\psi(x)\,.$$

Note that $\psi(x)$, unlike (4), represents here the solution of (8) (or (10)) at the last moment T of the pressure decrease.

Using (2) and (11), from (6), (7) we obtain

$$\uparrow \frac{\partial}{\partial t}\left\{\exp\left[-\varphi_2(x)\right]\exp\left[-\psi_2(x)(p_0-p)\right]+\xi_2(x)\exp\left(-\beta_{\text{liq}}(p_0-p)\right)\right\}=$$
$$=\chi_2\frac{\partial}{\partial x}\left\{\left[\exp\left[-\varphi_1(x)\right]\exp\left[-\psi_1(x)(p_0-p)\right]+\xi_1(x)\exp\left[-(\beta_{\text{liq}}-a_\mu)(p_0-p)\right]\right]\frac{\partial p}{\partial x}\right\}, \tag{12}$$

where

$$\xi_1(x)=\frac{k_{s0}}{k_0}\left[1-\delta_1\left(\psi(x)^{a_{ks}/\beta}\right]\theta(p_s-p_1)\,;\quad \xi_2(x)=\frac{m_{s0}}{m_0}\left[1-\delta_2\left(\psi(x)^{\beta_{ms}/\beta}\right]\theta(p_s-p_1)\,;$$

$$\chi_2=k_0/(m_0\mu_0)\,.$$

To estimate the change in the pressure in the regimes of the seam-pressure decrease and recovery, it is necessary to solve Eqs. (8) (or (10)) and (12) under respective initial and boundary conditions. These equations being nonlinear, it is expedient to solve them numerically.

Note that Eqs. (8) and (10) at $p \ge p_s$ are transformed to (3). Likewise, equations of the type (8), (10), and (12) can be derived for the plane-parallel and other cases.

To estimate the solutions of (8), (12), we formulate the following problem. Let, in a semi-infinite seam at the end $x = 0$, a constant speed of filtration w_0 be given. Initially, the pressure in the seam was constant, $p = p_0$. At the other end $x = \infty$ the pressure is kept in the initial state $p = p_0$. The above conditions can be given in the form

$$p(0,x)=p_0\,,\quad p(t,\infty)=p_0\,,\quad w(t,0)=w_0\,. \tag{13}$$

These conditions can be expressed in terms of φ as

$$\varphi(0,x)=1\,,\quad \varphi(t,\infty)=1\,,\quad \left.\left(\frac{\partial\varphi}{\partial x}-\lambda\varphi^{(\beta-\alpha)/\beta}\right)\right|_{x=0}=0\,, \tag{14}$$

where $\lambda = \beta q_0\mu_0/k_0p_0$ and q_0 is the mass flow per unit of area of the oil pool cross section (i.e., ρw_0).

After the operation of the oil pool till $t = T$ at the end $x = 0$ the condition that $w = 0$ is set, which corresponds to a shut-down (stop) of the well. Then the process of change in the seam pressure is investigated on the basis of Eq. (12). The initial and boundary conditions for this regime are of the form

$$p(0,x)=p_1\,,\quad p(t,\infty)=p_0\,,\quad \left.\frac{\partial p}{\partial x}\right|_{x=0}=0 \tag{15}$$

or in notation in terms of φ

$$\varphi(0,x)=\psi(x)\,,\quad \varphi(t,\infty)=1\,,\quad \left.\frac{\partial\varphi}{\partial x}\right|_{x=0}=0\,. \tag{16}$$

In conditions (15) and (16), $p_1(x)$ is determined by solving Eq. (8) (or (10)) under conditions (14) at time $t = T$.

To solve Eq. (8) under conditions (14) and Eq. (12) under conditions (15) (or (16)), we use the finite-difference method [14]. In the domain of $D = \{0 \le x < \infty, 0 \le t < T\}$, we introduce a mesh $\omega_{h\tau} = \{(x_i, t_j), i=\overline{0, I}, J=\overline{0, J}, x_i = ih, t_j = j\tau, h = L/I, \tau = T/J\}$, where L is some characteristic length of the seam, which is taken to be such that the boundary of the pressure-disturbed zone does not reach $x = L$. We denote the mesh solution corresponding to the point (x_i, t_j) by p_i^j, φ_j^i.

We shall first write Eq. (8) in the form

$$\downarrow \frac{\partial u(\varphi)}{\partial t} = \chi_1 \frac{\partial}{\partial x}\left[v(\varphi) \frac{\partial \varphi}{\partial x} \right], \tag{17}$$

where

$$u(\varphi) = \varphi + \frac{m_{s0}\theta(\sigma(\varphi))}{m_0}\left(1 - \delta_1 \varphi^{\beta_{ms}/\beta}\right)\varphi^{\beta_{\text{liq}}/\beta}; \quad v(\varphi) = \varphi^{\gamma-1} + \frac{k_{s0}\theta(\sigma(\varphi))}{k_0}\left(1 - \delta_2 \varphi^{a_{ks}/\beta}\right)\varphi^{-(a_\mu + \beta_{m0})/\beta}.$$

We approximate Eq. (17) on the mesh $\omega_{h\tau}$ to an accuracy of $O(\tau + h^2)$ by the explicit finite- difference scheme

$$\frac{u(\varphi_i^{j+1}) - u(\varphi_i^j)}{\tau} = \chi_1 \frac{1}{h}\left[a_{i+1}(\varphi_i^j) \frac{\varphi_{i+1}^j - \varphi_i^j}{h} - a_i(\varphi_i^j) \frac{\varphi_i^j - \varphi_{i-1}^j}{h} \right], \tag{18}$$

here

$$a_i(\varphi_i^j) = \frac{1}{2}\left[v(\varphi_{i-1}^j) + v(\varphi_i^j) \right].$$

From (18) we obtain mesh equations of the following form:

$$\varphi_i^{j+1} = V(\varphi_i^{j+1}) + F_i^j, \quad i = \overline{1, I-1}, \tag{19}$$

where

$$V(\varphi_i^{j+1}) = -\frac{m_{s0}}{m_0}\left[1 - \delta_1 (\varphi_i^{j+1})^{\beta_{ms}/\beta}\right](\varphi_i^{j+1})^{\beta_{\text{liq}}/\beta}\,\theta(\sigma(\varphi_i^{j+1}));$$

$$F_i^j = \frac{\chi_1 \tau}{h}\left[a_{i+1}(\varphi_i^j) \frac{\varphi_{i+1}^j - \varphi_i^j}{h} - a_i(\varphi_i^j) \frac{\varphi_i^j - \varphi_{i-1}^j}{h} \right] + \varphi_i^j + \frac{m_{s0}}{m_0}\left[1 - \delta_1 (\varphi_i^j)^{\beta_{ms}/\beta}\right](\varphi_i^j)^{\beta_{\text{liq}}/\beta}\,\theta(\sigma(\varphi_i^j)).$$

The initial and boundary conditions (14) are approximated as follows:

$$\varphi_i^0 = 1, \quad \varphi_i^{j+1} = 1, \quad i = \overline{0, I}, \quad \varphi_0^{j+1} = \frac{1}{3}\left(4\varphi_1^{j+1} - \varphi_2^{j+1}\right) - \frac{2\lambda h}{3}(\varphi_0^{j+1})^{(\beta-\alpha)/\beta}. \tag{20}$$

Equation (12) after the finite-difference approximation is reduced to an equation for p_i^j analogous to (19):

$$p_i^{j+1} = R(p_i^{j+1}) + G_i^j, \quad i = \overline{1, I-1}, \tag{21}$$

here

$$R(p_i^{j+1}) = p_i^{j+1} - \exp[-\varphi_2(x_i)]\exp[-\psi_2(x_i)(p_0 - p_i^{j+1})] - \xi_2(x_i)\exp[-\beta_{\text{liq}}(p_0 - p_i^{j+1})];$$

$$G_i^j = \frac{\chi_2 \tau}{h}\left[c_{i+1}(p_i^j) \frac{p_{i+1}^j - p_i^j}{h} - c_i(p_i^j) \frac{p_i^j - p_{i-1}^j}{h} \right] +$$

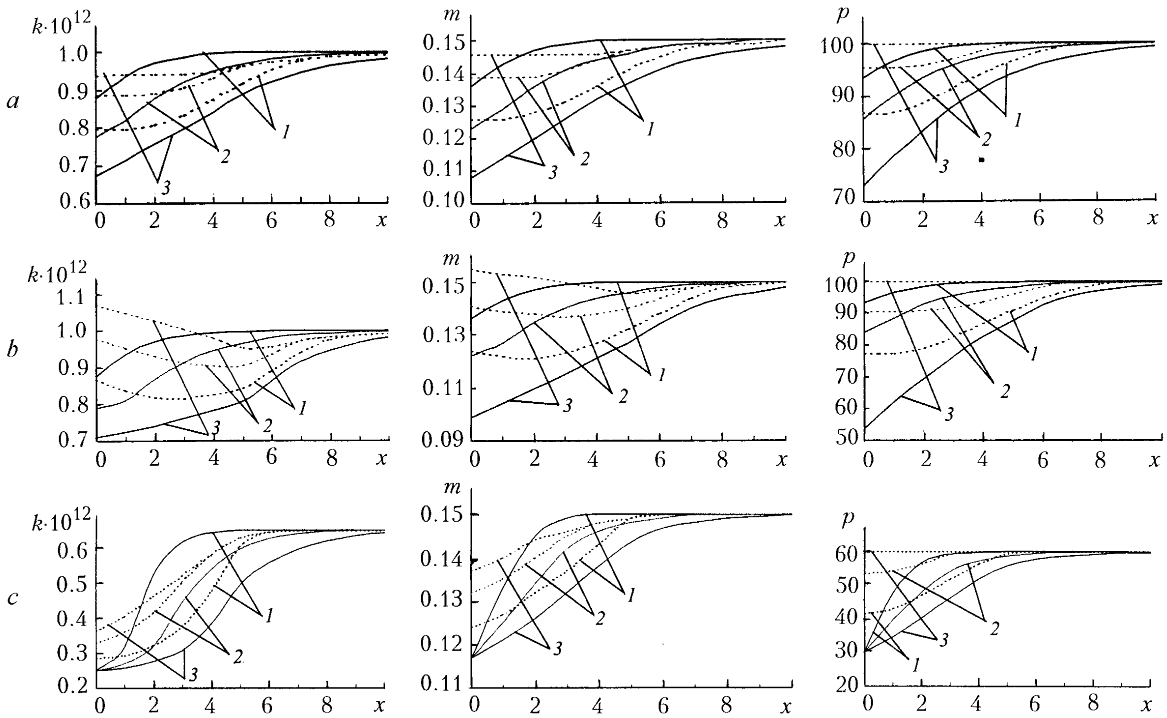

Fig. 1. Distribution of k, m, p in the regime of pressure decrease (solid lines) at t = 500 sec (1), 2000 (2), 4000 (3) and recovery (dotted lines) at t = 200 sec (1), 500 (2), 1000 (3): a) $k_{s0} = 0.4 \cdot 10^{-12}$ m^2, m_{s0} = 0.05; b) $k_{s0} = 0.75 \cdot 10^{-12}$ m^2, m_{s0} = 0.08; c) $k_{s0} = 0.28 \cdot 10^{-12}$ m^2, m_{s0} = 0.047, a_{ks} = 0.042 MPa^{-1}, β_{ms} = 0.0064 MPa^{-1}, Namangan oil field, well 13.

$$+ \exp [\varphi_2 (x_i)] \exp [- \psi_2 (x_i) (p_0 - p_i^j)] + \xi_2 (x_i) \exp [- \beta_{\text{liq}} (p_0 - p_i^j)] \,;$$

$$c_i (p_i^j) = \frac{1}{2} \left[r (p_{i-1}^j) + r (p_i^j) \right] ;$$

$$r (p_i^j) = \exp [- \varphi_1 (x_i)] \exp [- \psi_1 (x_i) (p_0 - p_i^j)] + \xi_1 (x_i) \exp [- (\beta_{\text{liq}} - a_\mu) (p_0 - p_i^j)] \,.$$

The initial and boundary conditions (15) are approximated as

$$p_i^0 = p_1 (x_i) \,, \quad p_0^{j+1} = (4 p_1^{j+1} - p_2^{j+1})/3 \,, \quad p_1^{j+1} = P_0 \,. \tag{22}$$

In the regime of pressure decrease, Eqs. (19) are solved under conditions (20), and in the regime of pressure recovery, Eqs. (21) are solved under conditions (22). The iteration method has been used.

For Eqs. (19) and (21) and the last condition of (20) the iteration process is constructed as follows:

$$(\varphi_i^{j+1})^{s+1} = V ((\varphi_i^{j+1})^s) + F_i^j \,, \quad (p_i^{j+1})^{s+1} = R ((p_i^{j+1})^s) + G_i^j \,, \quad i = \overline{1, I-1} \,,$$

$$(\varphi_0^{j+1})^{s+1} = \frac{1}{3} (4 \varphi_1^{j+1} - \varphi_2^{j+1}) - \frac{2 \lambda h}{3} [(\varphi_0^{j+1})^s]^{(\beta - \alpha)/\beta} \,,$$

where s is the iteration number, and proceeds until the conditions

$$|(\varphi_i^{j+1})^{s+1} - (\varphi_i^{j+1})^s| \le \varepsilon \,, \quad |(p_i^{j+1})^{s+1} - (p_i^{j+1})^s| \le \varepsilon \,, \quad i = \overline{0, I}$$

(ε is the accuracy of calculations) are satisfied.

In the calculations, we used the following input data: p_0 = 100 MPa, p_s = 90 MPa, μ_0 = 2.0 Pa·sec, L = 100 m, $k_0 = 10^{-12}$ m^2, m_0 = 0.15, ρ_0 = 950 kg/m^3, $w_0 = 3.5 \cdot 10^{-4}$ m/sec, $a_\mu = 5 \cdot 10^{-4}$ MPa^{-1}, $\beta_{liq} = 10^{-3}$ MPa^{-1}, a_{k0} = 0.02 MPa^{-1}, a_{ks} = 0.015 MPa^{-1}, β_{m0} = 0.015 MPa^{-1}, β_{ms} = 0.01 MPa^{-1}, η_k = 0.03 MPa^{-1}, and η_m = 0.02 MPa^{-1}.

Some of the results of the calculations are presented in Fig. 1. In the regime of pressure decrease, the dependences of k and m are similar in form to the usual dependences obtained without taking into account the particle carrying-out. In this regime, $p \le p_s$ at t = 2000 sec in the zone of $x \le 0.5$ m, and at t = 4000 sec this zone extends to $x \le 3$ m. We call it the seam-destruction zone; here the decrease in k and m with pressure is retarded. The pressure itself decreases not so intensively compared to the case where there is no carrying-out of particles. In the regime of pressure recovery, k and m increase but do not reach their initial values. Due to the seam destruction in the distributions of k and m in the vicinity of the hole the incomplete plastic recovery weakens.

Consider a situation where the role of the particle carrying-out is important. To this end, we have carried out calculations with $k_{s0} = 0.75 \cdot 10^{-12}$ m^2, m_{s0} = 0.08, and the values of the other parameters the same as before. The results of the calculations are presented in Fig. 1b. As is seen from the plot, in the vicinity of the well the formation of a strongly "washed" zone where the permeability and porosity at the end of the process of pressure recovery can have values exceeding the initial values is possible. This points to a more substantial influence of the particle carrying-out factor than of the plastoelasticity.

We shall use the above model for the real conditions of an oil field. On the basis of the results of hydrodynamical investigations of the wells at various depressions on the seam in different periods of seam-pressure decrease permeability- and porosity-pressure diagrams have been plotted. The diagrams are well described by dependences of the type (1) (1 — well 13; 2 — well 16; 3 — well 21).

$$\begin{aligned}
&1)\ k = 0.65 \cdot 10^{-12} \exp(-0.0647\,(60 - p))\,, \quad m = 0.15 \exp(-0.01\,(60 - p))\,,\\
&2)\ k = 0.35 \cdot 10^{-12} \exp(-0.0714\,(60 - p))\,, \quad m = 0.13 \exp(-0.015\,(60 - p))\,,\\
&3)\ k = 0.18 \cdot 10^{-12} \exp(-0.145\,(60 - p))\,, \quad m = 0.11 \exp(-0.02\,(60 - p))\,.
\end{aligned} \tag{23}$$

In the calculations, we used expressions (23) as well as the following values of the other initial parameters: p_0 = 60 MPa, p_w = 30 MPa, p_s = 50 MPa, ρ_0 = 950 kg/m^3, a_μ = 0.0005 MPa^{-1}, β_{liq} = 0.001 MPa^{-1}, η_k = 0.03 MPa^{-1}, η_m = 0.02 MPa^{-1}, and μ_0 = 2.0 Pa·sec. The values of k_{s0}, m_{s0}, a_{ks}, and β_{ms} for each well were taken differently. The results of the calculations for the conditions of well 13 are given in Fig. 1c. Analysis of the calculations shows that the irreversible changes in k and m on all wells are marked. On well 21, in the permeability dynamics nonmonotony has been obtained. Analysis of the influence of the factors of plastoelastic strain of the seam and carrying-out of particles separately shows that in the vicinity of the well the formation of a strongly "washed" zone, whose header properties are higher than those of a more remote zone, is impossible. In the latter zone, due to the plastoelastic strain of the header, k and m decrease appreciably and are not completely compensated by the particles being carried out.

Note that this technique can also be used to calculate the filtration indices for other operating conditions of an oil pool, in particular, where on x = 0 the fluid flow rate is given, the oil pool is closed, etc.

NOTATION

a_{k0}, coefficient of change in permeability, MPa^{-1}; a_{ks}, coefficient of change in k due to the particle carrying-out, MPa^{-1}; a_μ, coefficient of change in viscosity, MPa^{-1}; k, m and k_0, m_0, current and input (at $p = p_0$ = const) values of permeability (m^2) and porosity, respectively; k_{s0} (m^2) and m_{s0}, coefficients of the greatest possible increase in k and m due to the particle carrying-out; p, current pressure; p_w, pressure on the well bottom; p_s, pressure at which the seam integrity breaks; p_1, pressure distribution at the end of the pressure-decrease phase, MPa; t, time; T, maximum time of pressure decrease, sec; w_0 and w, constant and current speeds of filtration, m/sec; x, linear coordinate, m; β_{liq}, compressibility coefficient of liquid, MPa^{-1}; β_{m0}, coefficient of change in porosity, MPa^{-1}; β_{ms}, coefficient of change in m due to the particle carrying-out, MPa^{-1}; η_k and η_m, coefficients of irreversible change in permeability and porosity, MPa^{-1}; $\theta(x)$, unit Heaviside function; ρ and μ, ρ_0 and μ_0, current and initial (at $p = p_0$) density (kg/m^3) and viscosity (Pa·sec) of liquid, respectively. Subscripts: liq, liquid; w, well; s, sand.

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